MODELING & ANALYSIS

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A solution to the dynamical inverse problem of EEG generation using spatiotemporal Kalman filtering

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1. Introduction

This work deals with the problem of estimating the spatially extended sources of the electroencephalogram (EEG) from corresponding scalp recordings of the EEG, i.e. to solve the *inverse problem* of estimating the current distribution within human brain. We introduce a new approach for obtaining such solutions in a fully dynamical framework, using not only the EEG measurements at one instant of time (as it was the case with various previous algorithms, such as LORETA), but the full time series recorded simultaneously at all electrodes. The resulting *dynamical inverse solutions* are compared with LORETA solutions.

2. Method

In this study a discretisation of space into a set of $N_v = 3433$ *voxels* is employed (covering only cortical gray-matter areas); at each voxel a 3-dimensional time-dependent current vector is estimated. These 3*3433=10299 unobserved quantities are regarded as the *state of a dynamical system*, denoted by $\mathbf{J}(t)$. This state is assumed to evolve according to an autoregressive *dynamical equation*:

$$\mathbf{J}(t) = \mathcal{F}\big(\mathbf{J}(t-1), \mathbf{J}(t-2), \ldots, \mathbf{J}(t-p) \,|\, \boldsymbol{\vartheta}\big) + \boldsymbol{\eta}(t) \quad ,$$

where p denotes the positive integer model order, $\boldsymbol{\eta}(t)$ denotes a vector of dynamical noise and $\boldsymbol{\vartheta}$ denotes a vector of parameters.

The EEG time series $\mathbf{Y}(t)$ (as observed at $n_c = 18$ electrodes) results from an *observation equation*:

$$\mathbf{Y}(t) = \mathsf{K}\,\mathbf{J}(t) + \boldsymbol{\epsilon}(t) \quad ,$$

where K denotes a $n_c \times 3N_v$ transfer matrix (*lead field matrix*), which can approximately be modeled [2], and $\boldsymbol{\epsilon}(t)$ denotes a vector of observational noise.

The task of estimating $\mathbf{J}(t)$ from given $\mathbf{Y}(t)$ is interpreted as a high-dimensional state filtering problem. It can be approached by a new variant of *Kalman filtering* which allows to decompose an intractable high-dimensional filtering problem into a set of coupled tractable low-dimensional filtering problems. Each of these problems is centred at one voxel. The influence of all other voxels is regarded as an exogeneous variable which does not contribute to the state prediction and state estimation covariances of the Kalman filter.

BUT: The function $\mathcal{F}\big(\mathbf{J}(t-1), \mathbf{J}(t-2), \ldots, \mathbf{J}(t-p)$ is unknown.

Assumptions:

- $\mathcal{F}$ is linear.
- The model order p is at most 2.
- Direct interactions exist only between neighbouring voxels.
- $\boldsymbol{\epsilon}(t)$ is Gaussian with diagonal covariance matrix $\mathsf{C}_\epsilon = \sigma_\epsilon^2 \, \mathsf{I}_{n_c}$.
- $\boldsymbol{\eta}(t)$ is Gaussian with covariance matrix $\mathsf{C}_\eta = \sigma_\eta^2(\mathsf{L}^\dagger \mathsf{L})^{-1}$, where L is a discrete spatial 2nd-order derivative matrix.

Then the vector of unknown parameters $(\boldsymbol{\vartheta}, \sigma_\epsilon^2, \sigma_\eta^2)$ is of low dimension. These parameters can be estimated by minimisation of the Akaike Information Criterion (AIC), i.e. by a maximum-likelihood approach [3]:

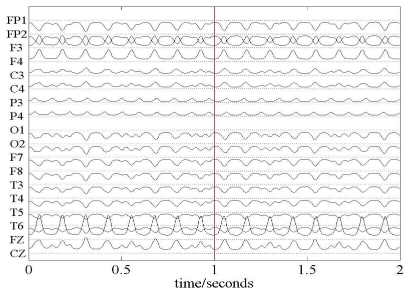
$$\begin{aligned}&\mathrm{AIC}(\boldsymbol{\vartheta}, \sigma_\epsilon^2, \sigma_\eta^2) = \\ &-2\textstyle\sum_{t=1}^{N_t}\big(\log|\mathsf{R}(t\,|\,t-1)| + \Delta\mathbf{Y}(t)^\dagger\,\mathsf{R}(t\,|\,t-1)^{-1}\,\Delta\mathbf{Y}(t) \\ &+ n_c\log(2\pi)\big) + 2\big(\dim(\boldsymbol{\vartheta}) + 2\big)\end{aligned}$$

where $\Delta\mathbf{Y}(t)$ denotes the observation prediction error, $|\mathsf{R}(t\,|\,t-1)|$ denotes the determinant of the observation prediction error covariance matrix and N_t denotes the length of the EEG time series.

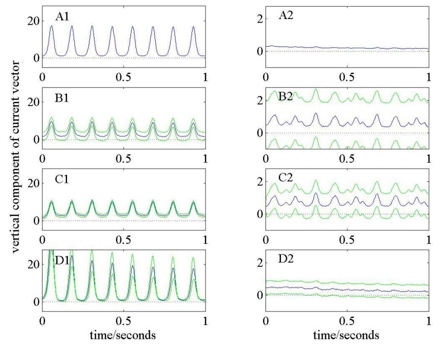
The state estimation covariance matrix of the Kalman filter naturally provides error estimates for the estimated currents $\hat{\mathbf{J}}(t|t)$; for LORETA [1] error estimates can be obtained from Bayesian arguments.

3. Results: Analysis of Simulated EEG

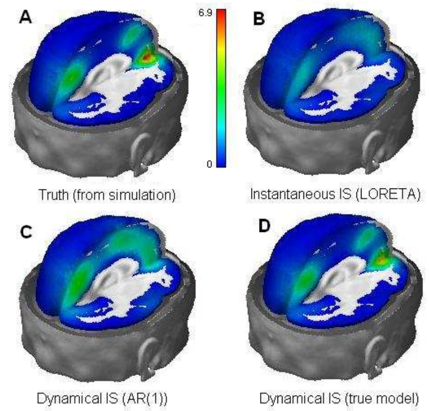
We design a simple deterministic spatiotemporal brain dynamics, driven by two sources of alpha-like oscillation. By multiplication with the lead field matrix we obtain simulated EEG:



The next figure shows the resulting inverse solutions for a voxel in right medial frontal gyrus (left) and for a voxel in left superior frontal gyrus (right). A1, A2 show the truth (i.e. the simulated sources); B1, B2 show the instantaneous inverse solution (LORETA) [1]; C1, C2 show the dynamical inverse solution using the simplest possible dynamical model for $\mathcal{F}$, an AR(1) model; and D1, D2 show the dynamical inverse solution using the true dynamical model. Blue lines: estimates; green lines: error intervals (95% confidence intervals).



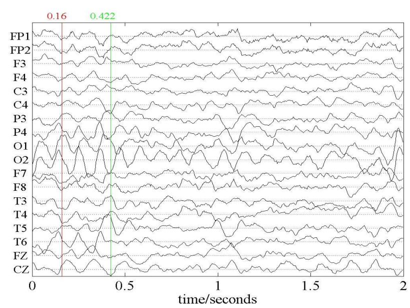
Next the spatial distribution of local current vector length at one instant of time (shown by a red line in the corresponding EEG figure) is shown for these four cases (IS = inverse solution):



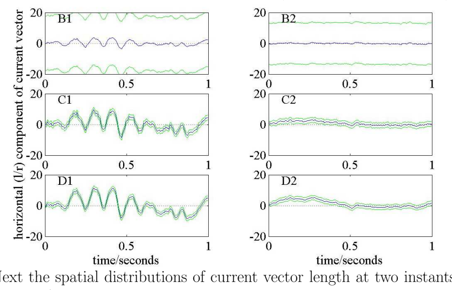
Result: If the true dynamical model is known (case D), the inverse solution is very similar to the truth (case A).

4. Results: Analysis of Clinical EEG

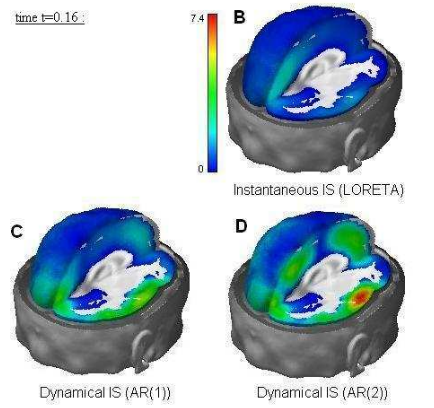
Two seconds from a clinical EEG (healthy male child, 8.5 years, awake, eyes closed) are chosen (sampling rate: 256 Hz); this data set was chosen in order to study the localisation of alpha activity:

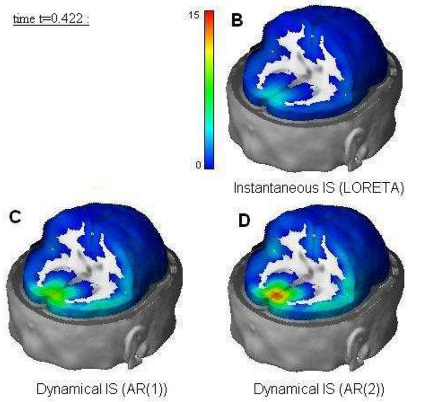


The next figure shows the resulting inverse solutions for a voxel in right cuneus (left) and for a voxel in right medial frontal gyrus (right). B1, B2 show the instantaneous inverse solution (LORETA); C1, C2 show the dynamical inverse solution using an AR(1) model for $\mathcal{F}$; and D1, D2 show the dynamical inverse solution using an AR(2) model. Blue lines: estimates; green lines: error intervals (95% confidence intervals).



Next the spatial distributions of current vector length at two instants of time (shown by a red and a green line in the corresponding EEG figure) are shown for these three inverse solutions:





Result: Given the same data and the same lead field matrix (such that no scaling effects have to be taken into account), the dynamical inverse solutions are finding a much more focussed source of alpha oscillation in the occipital area, as compared with the instantaneous inverse solution. Furthermore, considerable qualitative differences between these three inverse solutions can be seen also within other areas of brain.

Values of the Akaike Information Criterion (AIC) for these three inverse solutions [3]:

	AIC
LORETA	112328.4
dynamical IS, AR(1)-model	88990.1
dynamical IS, AR(2)-model	87131.3

5. Summary

- By introducing dynamical inverse solutions we have shown a systematic way to link the temporal aspect of EEG time series modeling with the spatial aspect of instantaneous inverse solutions.
- For this purpose a new spatiotemporal Kalman filter was designed, which is suitable to deal with high-dimensional state vectors.
- The use of the AIC (i.e. an estimator of Boltzmann entropy) enables the comparison of different dynamical inverse solution, thereby providing us with a systematic tool for developing spatiotemporal dynamical models [3].
- Both for instantaneous and dynamical inverse solutions the estimation error can be quantified. While error intervals for LORETA are repeatedly very large (rendering these inverse solutions insignificant), error intervals for dynamical inverse solutions have reasonable size.
- Spatiotemporal modeling yields spatially resolved estimates of dynamical noise, which may serve as a source of information about localised atypical behaviour in brain.
- Due to the nonlinear parameter estimation step this new technique is much more time-consuming than instantaneous techniques like LORETA [1].

6. References

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