# Directed Causality for Non-stationary Time Series Based on Akaike's Noise Contribution Ratio

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Abstract: Akaike's Noise Contribution Ratio (NCR) has been used for the analysis of causality of two-variable settings of biological time series in Neuroscience. In contrast to the conventional correlation definition, this methodology is able to detect the direction of the influence between two variables. However, if a third series intervention is taken into account, the validity of causality is questionable, since possible feedback with third series can induce spurious or indirect causality. In this paper, we introduce a modification to NCR that accounts for partial directed causality for the case of more than two variables (pNCR). We also extend this methodology for the case of non-stationary time series by means of the use of the sliding windows technique, which provides a time-frequency approach. This methodology produces a 2D matrix (time and frequency) of pNCR coefficients, which is difficult to interpret and visualize. To facilitate the visualization and interpretation of the pNCR for the case of non-stationary time series, we summarize the information of the

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spectrum of the pNCR as the area under the curve (pNCA), which projects this 2D matrix into the 1D space (a vector of coefficients), which shows the time course rate of influence from one variable to another in both directions.

## 1. Introduction

Detection of causal directions and their extent among a set of time series is one of the major interests in time-series analysis. The literature has centered on the causality analysis of two-variable case. If a third series intervention is taken into account, the validity of causality is questionable, since possible feedback with third series can induce spurious or indirect causality. For the purpose of causality analysis in multivariate time series data, Akaike (1968) proposed to decompose the power spectral density into components, each coming from an independent noise of multivariate autoregressive model (MAR). Akaike's noise contribution ratio (NCR) causality has been applied to ship engineering (Otomo *et al.*, 1972), nuclear power plant research (Fukunishi, 1977), neuroscience (Yamashita *et al.*, 2005, Wong and Ozaki, 2007) and physical science (Maki *et al.*, 2008). Unlike the more popular Granger causality (Granger, 1969) concept which focuses only in the time domain, NCR is a direct frequency domain definition.

Apparently without being aware of the earlier work by Akaike, directed coherence and directed transfer function were developed for modeling causality (Saito and Harashima, 1981, Kaminski and Blinowska, 1991). As a further step, Baccala and Sameshima (2001) proposed the concept of partial directed coherence for the conditional causality of one variable to another,given the presence of the other variables. All these more concepts have close links to the framework of Akaike causality; however, they focus mainly on the autoregressive (AR) coefficients and ignore the importance of noise variances, as both AR coefficients and noise variances play important roles in both Akaike causality and

Granger causality.

In this contribution we present a new approach for identifying direct and indirect causality. We do this by factoring the inverse of the cross spectral density matrix, similar to Akaike's NCR, which factors the cross spectral density matrix. The inverse of the cross spectral density matrix is analogous to the Fourier transform of the inverse cross-covariance function that contains conditional (partial) correlation between every pair of variables, given all the remaining variables. Once the cross spectral density matrix and its factors are calculated, we obtain the conditional causality ratio by taking the ratio of a factor with respect to the total. We will illustrate the new approach with numerical results of statistical time series modeling of simulated data.

## 2. Model

Given a set X = $\{x_n(t), n = 1, 2, \ldots, N\}$ of simultaneously observed jointly second-order stationary time series that are described by the $p^{\mathrm{th}}$ order MAR($p$) model:

$$[1] \qquad X(t) = \sum_{r=1}^{p} \Phi(r)X(t-r) + W(t)$$

where the autoregressive coefficients $\Phi(r)$ describe the linear relationship between time series $X(t)$ and $X(t$ - $r)$, and $W(t)$ represent the driving noise with Gaussian distribution with zero mean and diagonal variance $\Sigma$ with entries ${\sigma_1}^2$,${\sigma_2}^2$,..,${\sigma_N}^2$. Let

$$[2] \qquad \Phi(B) = \Phi(0) - \sum_{r=1}^{p} \Phi(r)B^r$$

so that Eq.[1] can be re-written as $\Phi(B)X(t) = W(t)$, where $B$ is a backshift operator such that $B^r X(t) = X(t$ - $r)$, and $\Phi(0)$ is an $N \times N$ identity matrix. Let also $P(f)$ be the cross spectral density matrix at frequency $f$.

[3] $$P(f) = \Phi(e^{-2i\pi f})^{-1}\Sigma(\Phi(e^{-2i\pi f})^H)^{-1}$$

[4] $$P(f)^{-1} = \Phi(e^{-2i\pi f})^H\Sigma^{-1}\Phi(e^{-2i\pi f})$$

where $A^H$ denotes the conjugate transpose of a matrix A. The $k$, $l^{\text{th}}$ entry of $\Phi$ $(e^{-2i\pi f})^H$ is given by $\phi_{lk}$ (0) - $\Sigma^p{}_{r=1}$ $\phi_{lk}(r)$ $e^{2ri\pi f}$ , $\phi_{lk}$ (0) = 1 if $k = l$ and 0 otherwise. We want to note that the subscript of $\phi$ is reversed and the exponential has positive power instead of negative because conjugate transpose operation has been applied to $\Phi$ $(e^{-2i\pi f})$. Hence, the $k^{\text{th}}$ diagonal entry of $P(f)^{-1}$ is given by

[5] $$\sum_{j=1}^{N}\left|\phi_{jk}(0) - \sum_{r=1}^{p}\phi_{jk}(r)e^{2ri\pi f}\right|^2 \frac{\pi}{\sigma_j^2}$$

and we define the partial noise contribution ratio (pNCR) from variable $k$ to variable $l$ conditioning on the remaining variables by:

[6] $$\frac{\left|\phi_{\ell k}(0) - \sum_{r=1}^{p}\phi_{\ell k}(r)e^{2ri\pi f}\right|^2/\sigma_\ell^2}{\sum_{j=1}^{N}|\phi_{jk}(0) - \sum_{r=1}^{p}\phi_{jk}(r)e^{2ri\pi f}|^2/\sigma_j^2}$$

We can explain causality between two variables not only at a specific frequency $f$ but also in a frequency range $(f_0, f_1)$ (pNCA). This can be done by integrating the nominator and denominator in Eq.[6]. Numerical integration such as Trapezoidal rule and Simpson's rule are simple approaches for this purpose.

## 3. Example 1: Simulation Study

### 3.1. Data

A first order ($p$=1) autoregressive system with 3 variables was simulated with a length of 6,000 instants of time. The dependency between the variables was set to vary in time as follows

$$\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = \begin{bmatrix} 1 & c_{12} & 0 \\ 0 & 1 & c_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.7 & 0.5 & 0 \\ 0 & 0.8 & 0.6 \\ 0 & 0 & 0.7 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \varepsilon(t)$$

where

$$\begin{bmatrix} c_{12} & c_{23} \end{bmatrix} = \begin{cases} \begin{bmatrix} 1 & 1 \end{bmatrix} & t = 1, \cdots, 1000 \\ \begin{bmatrix} 0 & 1 \end{bmatrix} & t = 1001, \cdots, 2000 \\ \begin{bmatrix} 1 & 0 \end{bmatrix} & t = 2001, \cdots, 4000 \\ \begin{bmatrix} 0 & 0 \end{bmatrix} & t = 4001, \cdots, 5000 \\ \begin{bmatrix} 1 & 1 \end{bmatrix} & t = 5001, \cdots, 6000 \end{cases}$$

The variances ${\sigma_1}^2$, ${\sigma_2}^2$ and ${\sigma_3}^2$ for each variable were set to 5, 10, and 20 respectively.

The lower triangular entries of the transition matrix are all zeros meaning $x_3$ is not caused by both $x_1$ and $x_2$, and $x_2$ is not caused by $x_1$. In Figure 1 we show the time series plot of the simulated data.

### 3.2. Results

By fitting a first order MAR model to the data, we obtain the coefficient estimates and hence calculate the pNCA shown in Figure 2. Among the three variables there are six distinct directions of causality. In each of the six panels, we show the true causality by a red line and the estimated causality by a blue line. The results show that the causality estimates are consistent.

Note that despite the fact that Variable 3 causes an indirect influence over Variable 1 (since Variable 3 influences Variable 2 and Variable 2 influences Variable 1) no direct causality is found between Variable 3 and Variable 1. In conventional correlation analysis a high correlation coefficient would be found between Variable 3 and Variable 1.

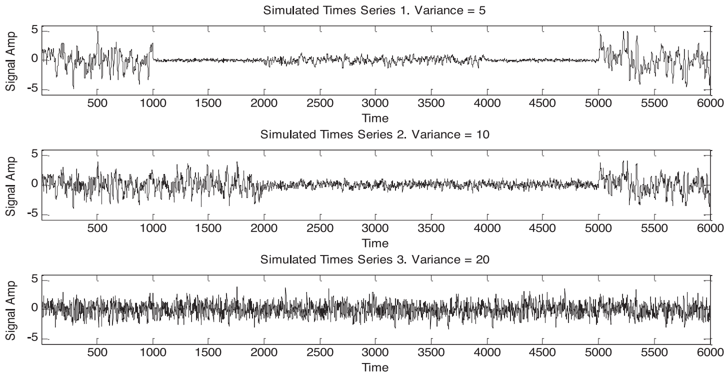


Figure 1. Simulated data.

Time series 1 does not influence any other time series. Series 2 influences 1 at specific time intervals and Series 3 influences 2 at some other time intervals. No direct influence from Series 3 to Series 1.

## 4. Example 2: 3–Way Human Interaction

### 4.1. Data

Three subjects were seated at a table with as shown in Figure 3. In their right hands they held a coin that they had to move to one of three positions in front of them, according to the experiment design. Each coin had a reflective marker (movement sensing) attached. One of the subjects was assigned as the Leader. Upon a sound signal, the Leader decides to move his coin to one of the three positions in front of him at random. The subject to the Leader's left then moves his coin to a different position than the Leader. It means that the second subject has two positions where to place the coin and he decides which of them at random. This subject is named the Sub-Leader. The third subject is the Follower and he has to place his coin in the remaining free position after the Sub-Leader moves his coin. When the Follower

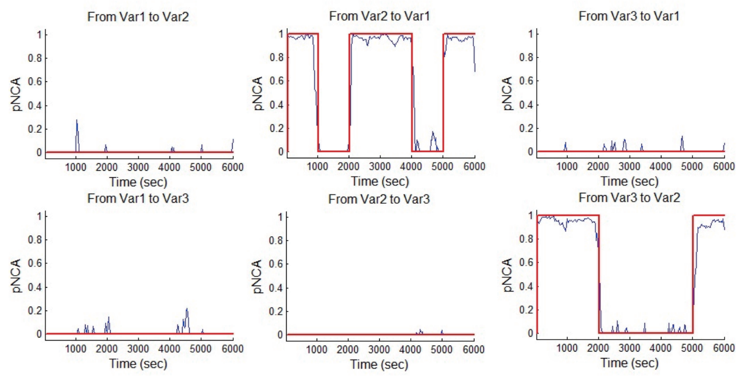


Figure 2. Simulation.

Red lines show the causal relationships simulated between the variables. Dependencies between variables were only simulated from variable 2 to variable 1 and from variable 3 to variable 2. Blue lines show the causality coefficients estimated by the pNCA. Note that pNCA has almost a perfect coincidence with the simulation and the ability of the method to detect that no direct causality exists from variable 3 to variable 1.

finishes his movement, the Leader moves his coin to another position at random and the cycle is repeated. Every 25 seconds, a sound signal indicates that the subjects have to change their roles. The Sub-Leader becomes the Leader, the Follower becomes the Sub-Leader and the Leader becomes the Follower. The situation is repeated 6 times, so each subject behaves as Leader, Sub-Leader and Follower two times.

In Figure 4 we show a cartoon of the causality pattern along the time, in which arrows indicate a causal relationship.

The position of the coin was acquired by 8 digital high speed infrared cameras (200 images/sec) (Oqus, Qualysis Inc., Gothenburg, Sweden), and Qualisys Track Manager software (QTM, Qualysis Inc., Gothenburg, Sweden) and exported as 3D data (XYZ). Due to the experimen-

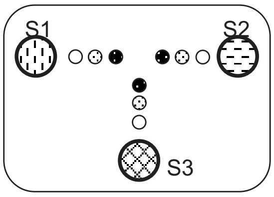


Figure 3. Schematic disposition of the subjects in the table.

Each subject has three positions in front and a coin in his right hand. A movement sensor is located in the coin. The camera system records the position of sensor each 200 msec.

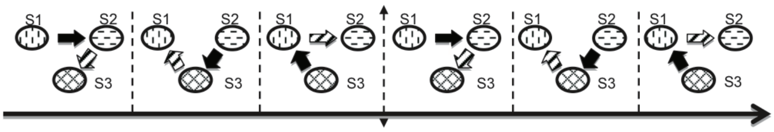


Figure 4. Schematic time course of the experiment.

The dashed vertical lines mark the sound signal indicating to change roles each 20 seconds. The dashed vertical line with arrows indicates the start of the second set of trials, where the subjects repeat their roles. The thick solid black arrows indicate Leader commanding Sub-Leader in each 20 seconds trial. The thick arrows with the diagonal bars indicate Sub-Leader commanding the Follower.

tal design, only the position in one of the axes is relevant to determine the position of the coin in time (X-axis for subjects 1 and 2 and Y-axis for subject 3). Thus, the values of the respective coordinates in time are used to create the three time series that constitute the input data for the pNCR procedure.

Figure 5 shows the measured data. This type of experiment generates such characteristic, which is very common in neurophysiological research. It appears very often in experiments like those which register head and eye motion, which are of great interest for our research. More complicated experimental settings require taking into account the position of the sensors in the 3D space, making more complex the causality analysis. In those cases, to create the time series data that will be used for the causality analysis, the components in X, Y and Z axis of the sensors have to be taken into account.

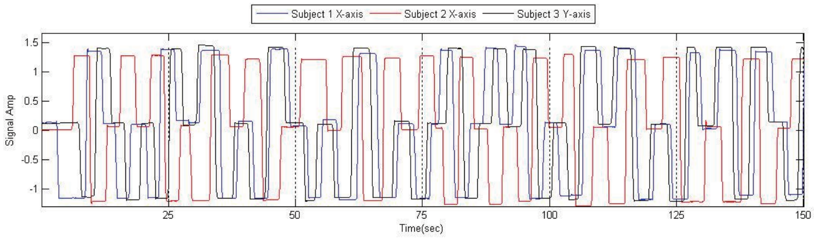


Figure 5. Normalized data time series: X-coordinates for subjects S1 and S2 and Y-coordinates for subject S3.

### 4.2. Results

We again perform a MAR modeling on a moving window of data to obtain the autoregressive coefficient estimates and the pNCR. In Figure 6 we show the causality coefficients between pairs of subjects. The results clearly show that causality is always one-way, which is consistent with the experimental setting. On the other hand, causality curves are going up when the subjects finished their leader role or going down when they started taking the leader role. Although there are indirect causal relationship between two subjects through the third subject, pNCR was able to partial out this indirect effect.

The apparent failure of pNCR to correctly gather the causal relationship between S1 and S2 in the last trial (from 125 to 150 seconds) is in fact a failure of S2 to follow S1 movement, as can be observed by carefully looking at S1 and S2 data in the last trial in Figure 5. Initiation of movement of S2 in this case is much delayed with respect to S1, and the method correctly captures and reports this desynchronization between the subjects.

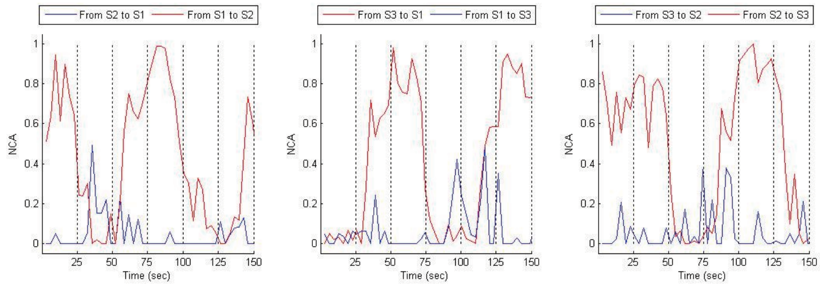


Figure 6. pNCA for the real data.

Left panel: time course causality between S1 and S2; central panel: between for S1 and S3; right panel: between S2 and S3. Note that the pNCA coefficients are in agreement with the experimental setting.

## 5. Conclusions

In this contribution we introduced a new method to detect conditional causality, as well as to differentiate it from indirect causality. The method is valid for both stationary and non-stationary time series data. By repeatedly performing MAR modeling on moving time windows, we can obtain time varying causality curves to interpret non-uniform causality pattern in the data.

Although the examples we presented are much different from forestry problems, we believe that the proposed method is still valid in those problems that interact each other with leader-follower characteristics. Such examples include the problem to identify dominant trees as leader in a forest stand and leading forest products in timber markets with large effects on other products. By knowing these, we could better predict market behavior or tree growth behavior. Application of the proposed method is general and not limited to the examples discussed here.

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